



**PERTH MODERN SCHOOL**  
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**Independent Public School**

## Course Specialist Test 4 Year 12

Student name: \_\_\_\_\_ Teacher name: \_\_\_\_\_

**Task type:**                      **Response**

**Time allowed for this task:**      40   mins

**Number of questions:**          7  

**Materials required:**         Calculator with CAS capability (to be provided by the student)

**Standard items:**              Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**Special items:**                Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available:**             44  marks

**Task weighting:**               10 %

**Formula sheet provided:** Yes

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

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Q1 (3 & 3 = 6 marks)

Solve the following.

a)  $\frac{dy}{dx} = \frac{3x-2}{y(5-y^2)}$  given that when  $x = 1, y = 1$ .

b)  $3x^4 \cos(2y) \frac{dy}{dx} = 10$  given that when  $x = 5, y = \pi$ .

Q2 (4 marks)

An iron has a temperature of  $54^\circ C$  is left in a room, of temperature  $18^\circ C$ , to cool such that the

temperature  $T^\circ C$  at time  $t$  minutes is given by  $\frac{dT}{dt} = k(T - 18)$ . After 15 mins the temperature of the

iron is  $37^\circ C$ . Determine the time taken for the iron's temperature to drop to  $22^\circ C$ .

Q3 (1, 5 & 2 = 8 marks)

The number  $N$  thousands, of bacteria cells living in a petri dish at time  $t$  hours is given by

$$\frac{dN}{dt} = 0.30N - 0.05N^2.$$

The initial number of cells was 2 thousand.

a) What is the limiting value of the number of cells as  $t \rightarrow \infty$ ?

b) Using calculus and partial fractions, show every step to express  $N$  in terms of  $t$ .

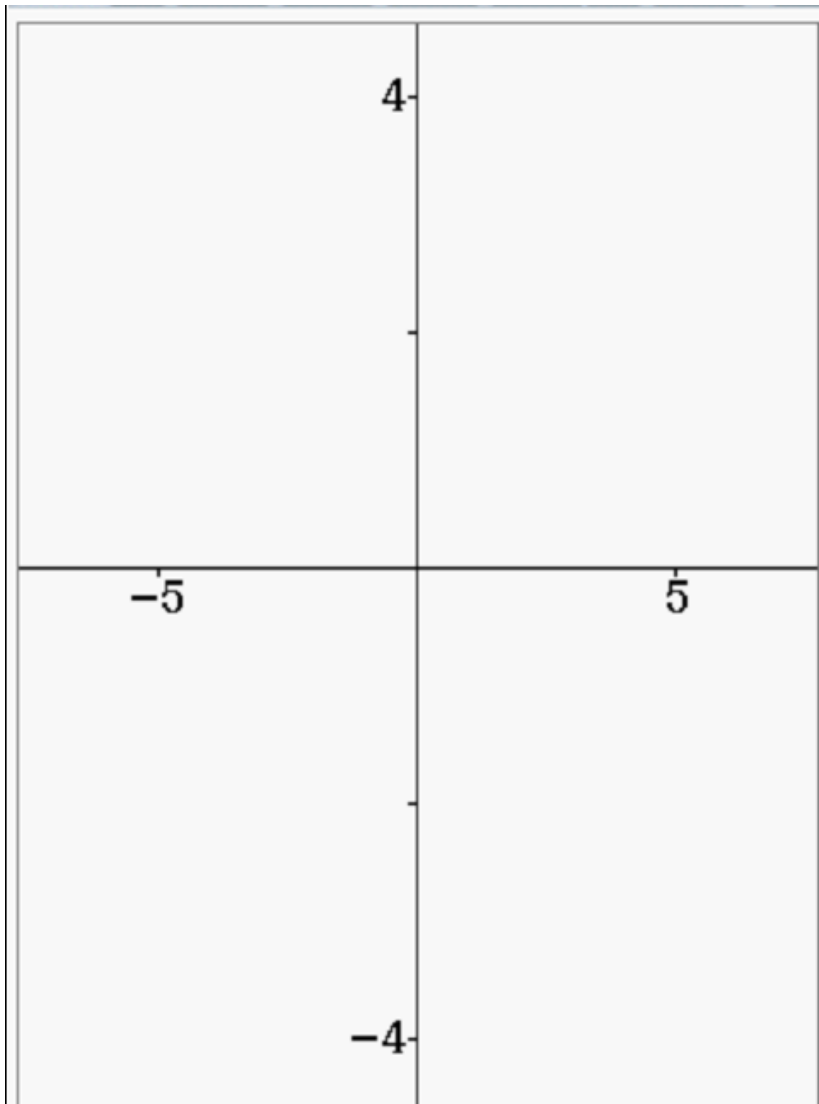
Q3-cont

c) Determine the number of cells after 15 hours.

Q4 (3, 2 &amp; 2 = 7 marks)

Consider the slope field  $\frac{dy}{dx} = (x-3)(x+2)$ 

a) Sketch this field on the axes below.



b) Draw the solution curve, axes above, that contains the point (1,1).

c) Determine the equation of the solution curve that contains (1,1).

Q5 (2, 2 & 3 = 7 marks)

Consider an object that is moving with Simple Harmonic Motion such that  $\ddot{x} = -9x$  with  $x, t$  in metres and seconds respectively. At  $t = 0$ ,  $x = 7$  metres and is at rest.

a) Determine a rule for  $x$  in terms of  $t$ .

b) Determine the exact speed when  $x = 3$  metres.

c) Determine the percentage of the time, to one decimal place, that the object is less than 3 metres from the mean position,  $x = 0$ .

Q6 (4 marks)

Consider an object that is initially at the origin and at rest such that its acceleration is given by

$\frac{dv}{dt} = \frac{1+v^3}{v} \text{ m/s}^2$  where  $v$  equals the speed in  $\text{m/s}$  at  $t$  seconds. Determine the exact speed when

its displacement from the origin is  $\ln(3)$  metres.

