

# PERTH MODERN SCHOOL

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## Course Specialist Test 4 Year 12

| Student name:                      | Teacher name:  |
|------------------------------------|--|
| Task type:                         | Response   |
| Time allowed for this task:40 mins |  |
| Number of questions:               | 7  |
| Materials required:                | Calculator with CAS capability (to be provided by the student)   |
| Standard items:                    | Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters                       |
| Special items:                     | Drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations |
| Marks available:                   | _44 marks  |
| Task weighting:                    | _10%   |
| Formula sheet provided:            | Yes  |

Note: All part questions worth more than 2 marks require working to obtain full marks.

#### Q1 (3 & 3 = 6 marks) Solve the following.

a) 
$$\frac{dy}{dx} = \frac{3x-2}{y(5-y^2)}$$
 given that when  $x = 1, y = 1$ .

b) 
$$3x^4 \cos(2y) \frac{dy}{dx} = 10$$
 given that when  $x = 5$ ,  $y = \pi$ .

Q2 (4 marks)

An iron has a temperature of  $54^{\circ}C$  is left in a room, of temperature  $18^{\circ}C$ , to cool such that the temperature  $T^{\circ}C$  at time *t* minutes is given by  $\frac{dT}{dt} = k(T-18)$ . After 15 mins the temperature of the iron is  $37^{\circ}C$ . Determine the time taken for the iron's temperature to drop to  $22^{\circ}C$ .

### Q3 (1, 5 & 2 = 8 marks)

The number N thousands, of bacteria cells living in a petri dish at time t hours is given by  $\frac{dN}{dN} = 0.30N - 0.05N^2$ 

 $\frac{dN}{dt} = 0.30N - 0.05N^2.$ 

The initial number of cells was 2 thousand.

a) What is the limiting value of the number of cells as  $t \rightarrow \infty$ ?

b) Using calculus and partial fractions, show every step to express N in terms of t.

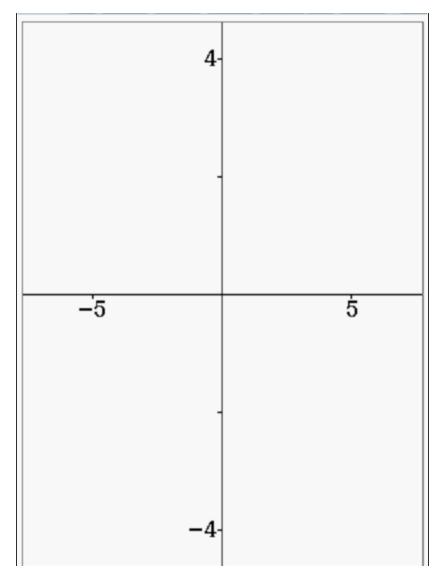
Q3-cont

c) Determine the number of cells after 15 hours.

Q4 (3, 2 & 2 = 7 marks)

Consider the slope field  $\frac{dy}{dx} = (x-3)(x+2)$ 

a) Sketch this field on the axes below.



- b) Draw the solution curve, axes above, that contains the point (1,1).
- c) Determine the equation of the solution curve that contains (1,1).

#### Q5 (2, 2 & 3 = 7 marks)

Consider an object that is moving with Simple Harmonic Motion such that  $\ddot{x} = -9x$  with x, t in metres and seconds respectively. At t = 0, x = 7 metres and is a rest.

- a) Determine a rule for x in terms of t.
- b) Determine the exact speed when x = 3 metres.
- c) Determine the percentage of the time, to one decimal place, that the object is less than 3 metres from the mean position, x = 0.

Q6 (4 marks)

Consider an object that is initially at the origin and at rest such that its acceleration is given by  $\frac{dv}{dt} = \frac{1+v^3}{v}m/s^2$ where *v* equals the speed in *m/s* at *t* seconds. Determine the exact speed when its displacement from the origin is  $\ln(3)$  metres.

### Q7 (2, 3 & 3 = 8 marks)

A lolly company makes jelly beans where the mass of one jelly bean is normally distributed with a mean of 23.4 mg and a standard deviation of 3.2 mg. (Note: 1g=1000mg)

a) Determine the probability to two decimal places that the total mass of 85 jelly beans is more than two grams.

b) Given that the probability that the mean mass of a jelly bean differs from the population mean by more than 0.35 mg is 5%, determine n, the number of jelly beans that need to be sampled.

c) On a particular day the operator of a machine that makes jelly beans is suspected of being faulty. A sample of 200 jelly beans had a sample standard deviation of 3.8 mg with a total mass of 5.4 grams. Present a mathematical argument to either support or to dismiss such a claim.